University Center for Mathematical Modeling, Applied Analysis and Computational Mathematics

Semester Seminar, 19$^{\text{th}}$ December 2016, 7:55–12:10, Room K3

Schedule

Time	Speaker	Title
7:55		Opening
8:00	Martin Lanzendörfer	Multiple solutions to the steady flow of incompressible fluids subject to constant traction or do-nothing boundary conditions on artificial boundaries
8:20	Iveta Hnětynková	Tensor generalizations of the total least squares problem
8:40	Marek Cúth	"Lipschitz-free" spaces over $\mathbb{R}^n$
9:00	Michal Pavelka	Hamiltonian evolution of complex systems and thermodynamic optimization
9:20		Coffee break
9:30	Ondřej Souček	Plume activity and tidal deformation on Enceladus influenced by faults and variable ice shell thickness
9:50	Vít Průša	On thermodynamics of viscoelastic rate type fluids with temperature dependent material coefficients
10:10	Miloslav Vlasák	Stability of the ALE space-time discontinuous Galerkin method for nonlinear convection-diffusion problems in time-dependent domains
10:30		Coffee break
10:45	Ondřej Kurka	On the complexity of the isomorphism class of a Banach space
11:05	Benjamin Vejnar	The relation of being homeomorphic for continua is as complicated as for compacta
11:25	Dušan Pokorný	Minkowski measurability of the images of self similar fractals
11:45	Václav Vlasák	Hulls of Haar meager sets
12:05		Conclusion

Abstracts

Martin Lanzendörfer: Multiple solutions to the steady flow of incompressible fluids subject to constant traction or do-nothing boundary conditions on artificial boundaries. It seems natural to our minds to separate the "inner" and "outer", despite of them being inseparable in essence. In the (computational) fluid dynamics it is often practical to consider the flow problems in a bounded domain. In many cases, certain parts of the domain's boundary are artificial: not related to any natural physical interface. The choice of boundary conditions to be imposed on artificial boundaries is then a modelling issue, where neither the physics alone nor the mathematical analysis alone give conclusive recommendations.

The talk will focus on the inflow and outflow boundary conditions for Navier-Stokes equations in the case that the flow rate is not known a priori. We will discuss a particular set of examples demonstrating the non-uniqueness and the (lack of) stability of steady solutions for the "do-nothing" b.c.'s and the b.c.'s based on prescribed constant traction.

Iveta Hnětynková: Tensor generalizations of the total least squares problem. The total least squares (TLS) is a widely used linear data fitting approach for solving problems $\mathbb{A}\mathbb{X} \approx \mathbb{B}$, where the errors are present both in the model matrix $\mathbb{A}$ and the observation matrix $\mathbb{B}$. While the (ordinary) least squares method has already been generalized to multilinear problems, where the model and/or the observations are in a tensor form, this is to our knowledge not the case for the TLS. This contribution proposes possible tensor TLS definitions and investigates their properties including relations to the linear TLS formulation.

Marek Cúth: "Lipschitz-free" spaces over $\mathbb{R}^n$. Given a metric space M, it is possible to construct a Banach space $\mathcal{F}(M)$ in such a way that the lipschitz structure M corresponds to the linear structure of $\mathcal{F}(M)$. The space $\mathcal{F}(M)$ is sometimes called "Lipschitz-free space". The study of those spaces became an active field of research (mostly in France). Even though it is easy to define Lipschitz-free spaces over separable metric spaces, not much is known about their structure - for example, it is not known whether $\mathcal{F}(\mathbb{R}^2)$ is isometric with $\mathcal{F}(\mathbb{R}^3)$.

During my talk I will present a recent isometric characterization of $\mathcal{F}(\mathbb{R}^n)$, which was obtained in a collaboration with Ondřej Kalenda and Petr Kaplický. Essential part of our proof was to use certain knowledge from the theory of partial differential equations, which was the role of Petr Kaplický in the project. Hence, this result in a way belongs to the border of the theoretical and applied science and it agrees with the aim of the UNCE Math MAC project.

Michal Pavelka: Hamiltonian evolution of complex systems and thermodynamic optimization. We present two topics. Firstly, new insight was brought into Hamiltonian modeling of evolution equations of natural phenomena. In particular, a hierarchy of Poisson brackets was identified [3], within which the reversible evolution of (extended) kinetic theory and non-Newtonian fluids can be rigorously derived in a unified manner. The Hamiltonian approach to fluid dynamics was then used to address the issue of possible extra mass flux in hydrodynamics [2], where fluids undergoing rapid compression were shown to exhibit such an extra mass flux. The second topic is related to the practical problem of optimization of electricity producing devices, e.g. fuel cells. It was explicitly shown in [1] that the standard optimization criterion - entropy production - fails when the device is in non-isothermal conditions.

[1] Vágner, P., Pavelka, M., Maršík, F., Pitfalls of exergy analysis, arXiv:1611.01410, Accepted to Journal of Non-Equilibrium Thermodynamics (2016)

[2] Ván, P., Pavelka, M. and Grmela, M., Extra mass flux in fluid mechanics, http://arxiv.org/abs/1510.03900v4, Accepted to Journal of Non-Equilibrium Thermodynamics (2016)

[3] Pavelka, M., Klika, V., Esen, O. and Grmela, M., A hierarchy of Poisson brackets in non-equilibrium thermodynamics, arXiv:1512.08010, Physica D: Nonlinear Phenomena 335 (2016), pages 54–69

Ondřej Souček: Plume activity and tidal deformation on Enceladus influenced by faults and variable ice shell thickness. The south polar region of Saturn's moon Enceladus has been subjected to a thorough scientific scrutiny since the Cassini mission discovery of an enigmatic system of fractures informally known as "tiger stripes". This fault system is possibly connected to the internal water ocean and exhibits a striking geological activity manifesting itself in the form of active water geysers on the moon's surface. We investigate the role of variations in the ice shell thickness and of the "tiger stripe" fractures crossing Enceladus' south polar terrain on the moon's tidal deformation by performing finite element calculations in three-dimensional geometry. Combination of thinning in the polar region and the presence of faults has a synergetic effect leading to an increase of both the displacement and the stress in the south polar terrain by an order of magnitude compared to the traditional model with a uniform shell thickness and without faults. Assuming a simplified conductive heat transfer and neglecting the heat sources below the ice shell, we compute the global heat budget of the ice shell. Assuming that the inelastic properties of the shell are described by a Maxwell viscoelastic model, we show that unrealistically low average viscosity of the order of $10^{13} Pa\, s$ is then necessary for preserving the volume of the ocean, suggesting the important role of the heat sources in the deep interior. Similarly low viscosity is required to predict the observed delay of the plume activity which hints at other delaying mechanisms than just the viscoelasticity of the ice shell. The presence of faults results in large spatial and temporal heterogeneity of geysering activity compared to the traditional models without faults providing a criterion to discriminate between the different structural models of the tiger stripes.

Vít Průša: On thermodynamics of viscoelastic rate type fluids with temperature dependent material coefficients. We derive a class of thermodynamically consistent variants of Maxwell/Oldroyd-B type models for viscoelastic fluids. In particular, we study the models that allow one to consider temperature dependent material coefficients. This naturally calls for the formulation of a temperature evolution equation that would accompany the evolution equations for the mechanical quantities. The evolution equation for the temperature is explicitly formulated, and it is shown to be consistent with the laws of thermodynamics and the evolution equations for the mechanical quantities. The temperature evolution equation contains terms that are ignored or even not thought of in most of the works dealing with this class of fluids. The impact of the additional terms in the temperature evolution equation on the flow dynamics is documented by the solution of simple initial/boundary value problems.

Miloslav Vlasák: Stability of the ALE space-time discontinuous Galerkin method for nonlinear convection-diffusion problems in time-dependent domains. We will consider the heat equation in time-dependent domains. This problem will be discretized by space-time discontinuous Galerkin method with the aid of Arbitrary Lagrangian Eulerian (ALE) formulation. For this problem we will show the stability of the scheme. Finally, the ideas will be employed for nonlinear convection-diffusion problems.

Ondřej Kurka: On the complexity of the isomorphism class of a Banach space. In my previous talk, I presented a result on the descriptive complexity of some classes of separable Banach spaces. This time, the isomorphism classes will be discussed. I will mention some known results and some ideas how to solve the main question in this area.

Benjamin Vejnar: The relation of being homeomorphic for continua is as complicated as for compacta. Complete classification of mathematical structures up to isomorphism is usually possible only in some special cases. There is for example no satisfactory way to classify all groups up to isomorphism or compact metric spaces up to homeomorphism. In spite of that, one can compare the complexity of the isomorphism relation of some structures with respect to some other.

The idea is as follows. First we suppose that all the structures are represented by points in some topological (usually Polish) space. This can be often done in some natural way. Borel sets and Borel mappings in Polish spaces are supposed to be simple for our purposes. We compare the complexity of equivalences E on a space X and F on a space Y in the following way. E is said to be Borel reducible to F if there is a Borel map $f : X \to Y$ such that $aEb \iff f(a)Ff(b)$ for every a, b in X. In this case E is viewed to be simpler than F.

Zielinski proved recently that the homeomorphism equivalence relation of compacta is Borel reducible to the universal orbit equivalence relation and vice versa. No matter what universal orbit equivalence relation is, its complexity is already well understood. A similar result was given by Chang and Gao for continua. We were able to simplify their argumentation substantially.

Dušan Pokorný: Minkowski measurability of the images of self similar fractals. As a follow-up to my talk from the last meeting, I will present our recent progress in understanding of the asymptotics of the counting functions on s-regular sets with the main result being the extension of our methods to the most important case of the Minkowski content. I will try to explain why this case is important and what is interesting about the method of the proof itself. This talk will be based on a joint work with M. Rauch.

Václav Vlasák: Hulls of Haar meager sets. Many classes of small sets were studied in mathematics. Well known examples of small sets are null sets in Euclidean spaces and their topological counterpart meager sets. Christensen introduced a notion of Haar null sets as a generalization of null sets in Polish (separable and completely metrizable) groups. In a similar way, Darji introduced Haar meager sets as their topological counterpart. I remind these notions and mention some new results. Namely, that there exist Haar meager sets without ”nice” hulls and that compact subsets of some nonlocally compact Polish groups are Haar meager.